Criteria for knowing a geometrical object
-the enactivist perspective-

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Structure of this presentation

Introduction –the enactivist approach

The objective of this article

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the key idea *criterion of distinction*

Theoretical framework(2)

the world brought forth to know a geometrical object

Methodology –the qualitative method

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student's 3 criteria for knowing objects

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closed, open situation

Conclusion

Introduction

- Enactivism **<*Knowing* is a dynamic, adaptive action.>**

  -A classroom is a kind of <u>mathematical working space</u> involving a teacher, students and an environment …

- "viability" leads to "**bringing forth the distinct worlds of significance**"(Proulx & Simmit, 2013, 2016)

  -differentiating as an object (concrete/conceptual), bringing forth a world for the object, co-evolving, …

  *counting, shaping, measuring, … are based on differentiation, and then, 'emphasizes' on the distinct worlds according to each activity.

  -a mathematical object: generated in the world and co-evolving with every actor and an environment

  *but how??

The objective of this article

What are the criteria needed by students to come to know <u>a geometrical object</u> ?

[image-definition-figural concept]

(Fischbein, 1993)

RQ1: What criteria do students adopt to generate an object and determine whether it is geometrical or not?

→ theoretical, possible, actual knowing

RQ2: What factors are important to initiate and evolve interactions between the teacher, the students, and the learning environment?

→ a closed, open problem situation

Theoretical framework(1)

-the key idea behind *enactivism*

≪ The act of indicating any being, thing, or unity involves making ***an act of distinction*** which distinguishes what has been indicated as separate from its background. ≫ (Maturana & Varela, 1992)

- Criterion of distinction for a geometrical object: **visually** and **in language** (Simmt & Kieren, 2015)

For a detailed explanation to form a geometrical object with social dimension …

- **Isomorphism** (Greer & Harel, 1998) inherent in the distinct worlds that emerge through an evolutionary, social process

Why isomorphism ?

on a paper

"this is a triangle"

→(re)forming an environment for actors

a description

"a triangle is …"

action with

manipuratives in

physical space

https://www.tokyo
shoseki.co.jp/polydron/
what is this?

on a computer screen

A geometrical object is <u>embodied</u> by drawing, constructing and so on, and then <u>recognized the 'uniformity'</u> such as *invariance*

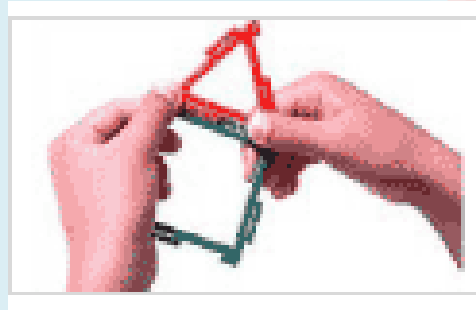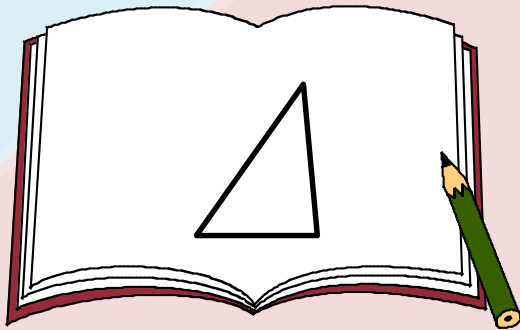ex. number of points and its position relation between all embodied objects through some operations in a classroom.

Theoretical framework(2)

-the world to know <u>a geometrical object</u>

"<u>If</u> I follow the condition, <u>then</u> I can see a lot of figures!"

[the geometrical world]

[the physical world]

*<u>Objects are mediated</u> by theoretically thinking about the creation of them as a model.

*<u>Objects could exist</u> under the physical law.

The way of knowing and doing depends on the world that is brought forth. (Proulx & Simmt, 2013)

Methodology

- **The qualitative research method** (Flick, 2009)
- **Data for analysis**

  A 3ʳᵈ grade lesson, collaboratively designed

  <main activity>

  1. constructing figures by using several tools:
  composing concrete objects, drawing, talking about, …

  2. geometrically and logically demonstrating the figures: **"Find a triangle, and is it a triangle definitely?"**

  ✓ <u>video data</u> and <u>a transcript</u> of the lesson, <u>a field note</u> by the author, <u>student's worksheets</u>

  ✓ <u>post discussions</u> with the teacher

Results and discussions(1)

[activity 1]

Fig.1 Touch gesture

Although the teacher showed the process of connecting the same length sides …

"I am not sure that this angle is same as this one because this is hidden by the teacher's hands."

[actual knowing]
Bodily actions and physical sensations are preferred to convince oneself of existence and truth/false.
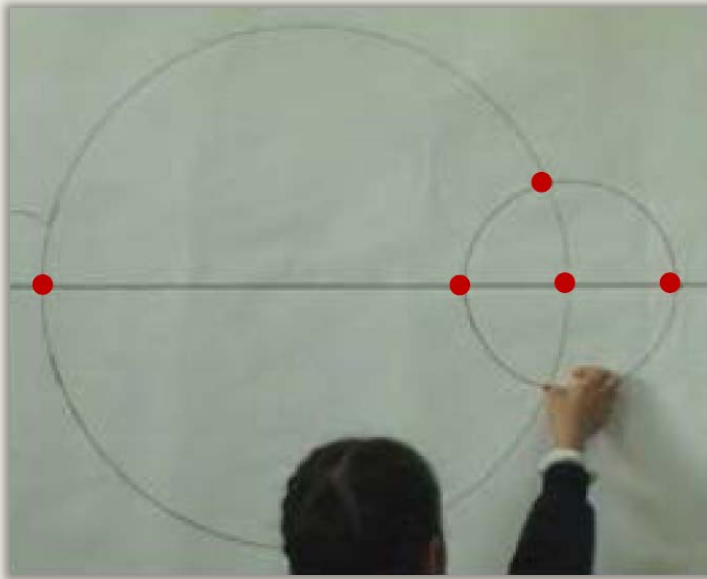
[activity 2]

Fig.2 Embodied points

(a) drawing a certain line;
(b) drawing a large circle;
(c) confirming that **some points are generated as intersections**;
(d) putting small circles on.

No points exist initially on the Fig.2 diagram in a physical sense, because these are expressed as filled circles on a textbook.

[actual knowing]

However, the teacher put a red mark on a intersection;
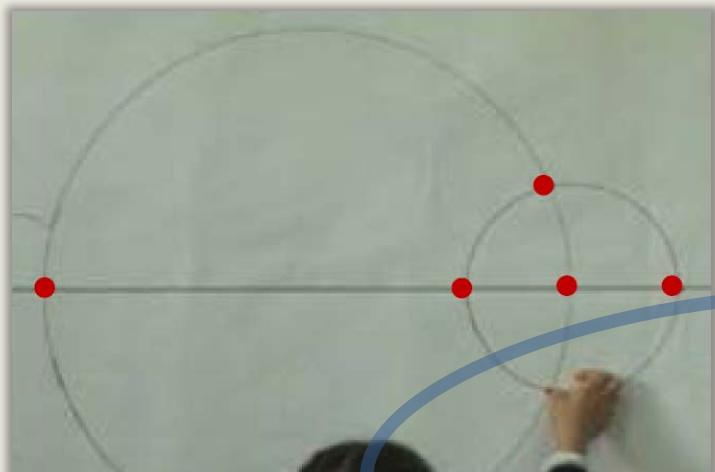
"Now I see."

"I can see some points."

in a linguistic sense.

[possible knowing]
   Definition is used as a method to generate possible figures.

[Red marks on a whiteboard is a certain, possibly existing object for the students.]

[activity 2]

[activity 3]

[possible knowing]
Definition is used as a method to generate possible figures.

[theoretical knowing]
Definition is used as a method to confirm the existence of the figures.

Fig.2 Embodied points

Fig.3 Generating a possible triangle

...concerning about S's reasoning whether the triangle is isosceles or not (Fig.3)

"All radii have the same length. Well, two points are connected, and the exterior connected parts are the same, so …"
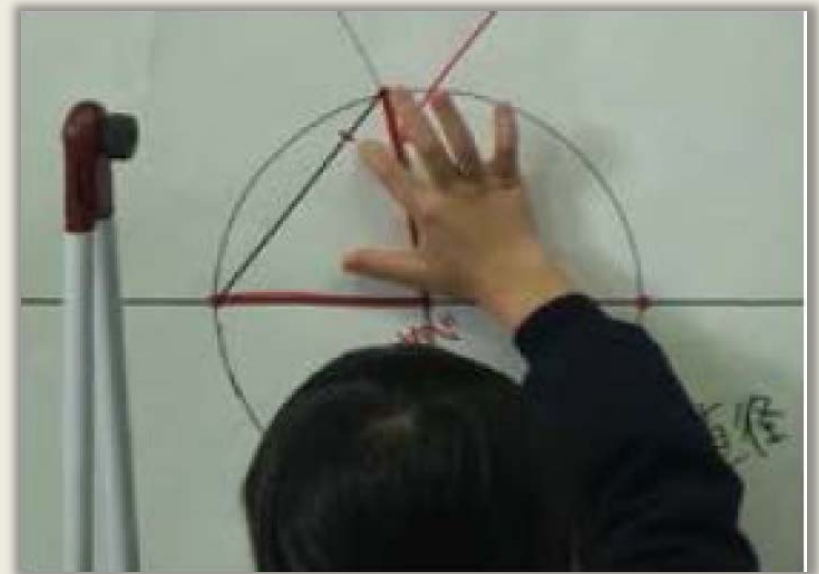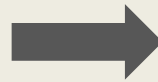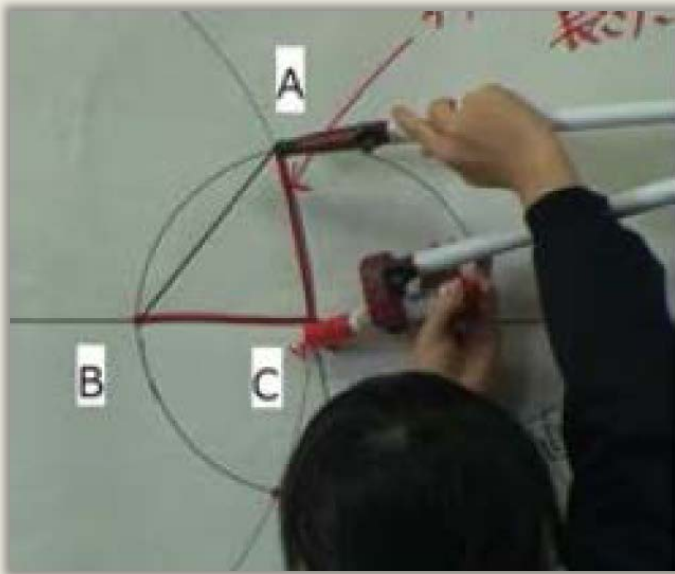
Fig.4 Demonstration by measuring and comparing

[theoretical knowing]

… concerning student Y's reasoning that the triangle is not regular by "measuring" (Fig.4)

"The length of this reaches here and the black one is left, so it is an isosceles triangle."
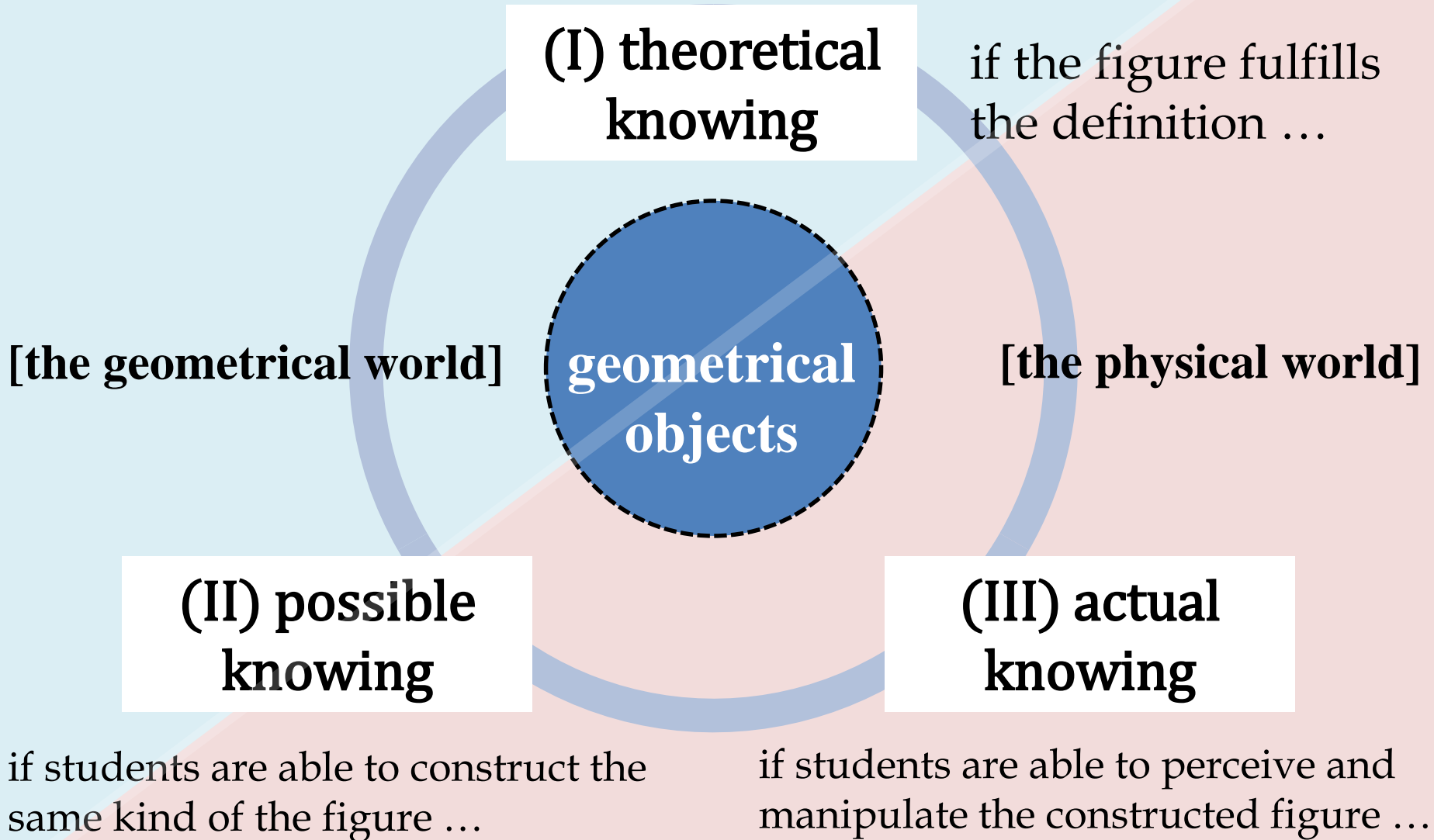
[actual knowing]

"I don't think that it is a regular triangle."

Results and discussions(1)

(pp.*-)

the individual triangle was known from its relationships with the given line and circles, which was realized based on the specific geometrical definitions and properties in **the geometrical world**,

and was actualized by drawing and constructing it in **the physical world**, all of which evolved through the interactions between the object and its background.

Students' 3 criteria to know geometrical objects

(I) theoretical knowing

if the figure fulfills the definition …

[the geometrical world]

geometrical objects

[the physical world]

(II) possible knowing

(III) actual knowing

if students are able to construct the same kind of the figure …

if students are able to perceive and manipulate the constructed figure …

Results and discussions(2)

An open situation is the trigger
for evolving interactions.

The endorsements of object's existence:
- ✓ perception, action, manipulation
- ✓ recognizing a relationship with the given line and circles and a rule to generate figures
- ✓ languaging, defining, …

Fig.2 and 3 [open situations] allow students to act both physically and theoretically, that enhance classroom discussions about object's existence.

Rough sketch for conclusion

the multi-world **brought forth with emergent objects**

closed   open
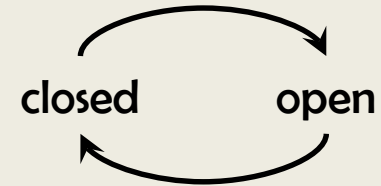
environment

<acting on>

inter-actions
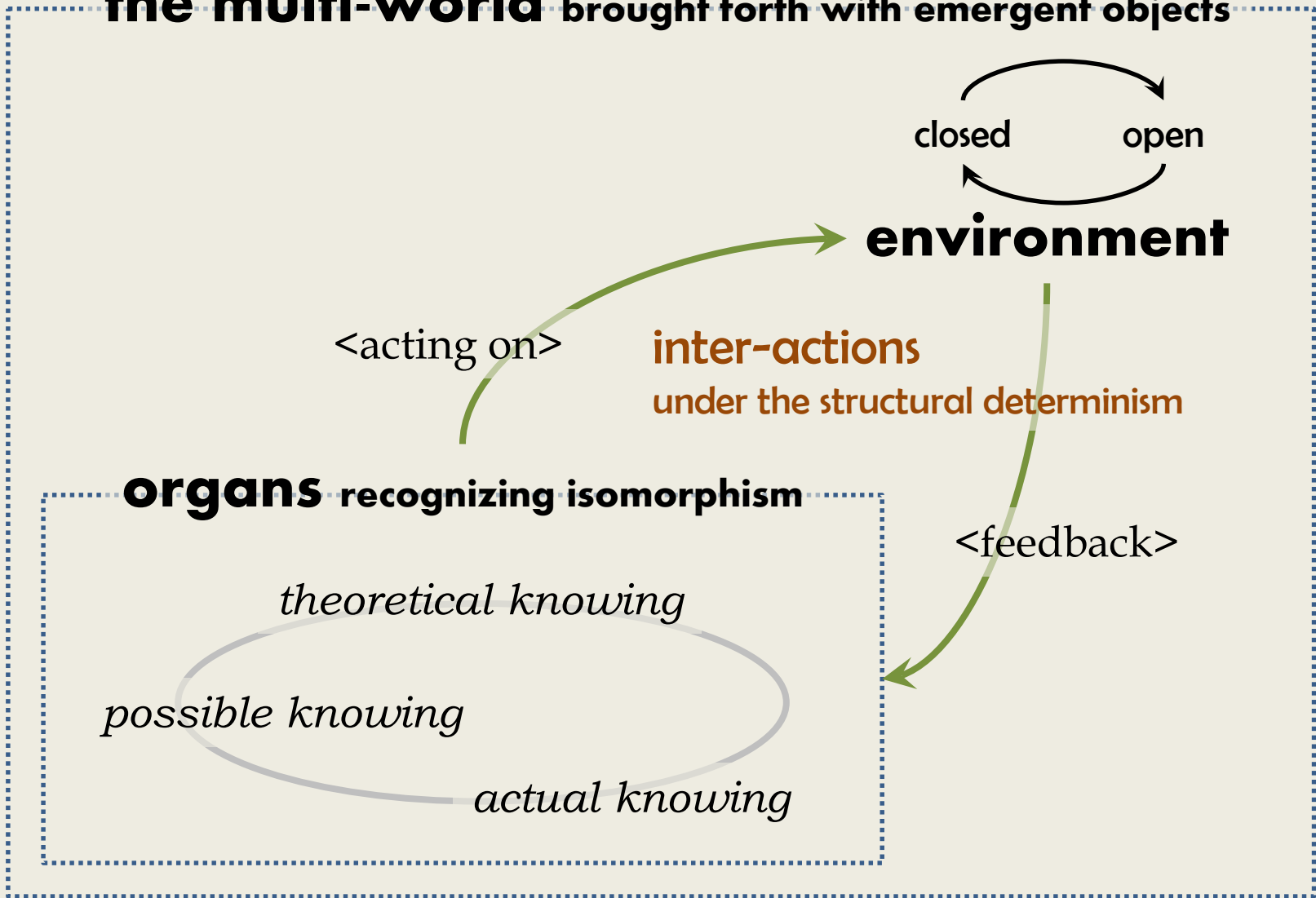**under the structural determinism**

organs **recognizing isomorphism**

<feedback>

theoretical knowing

possible knowing

actual knowing

Conclusion

The article identified at least 3 STUDENTS' criteria for knowing geometrical objects:
(I)theoretical, (II)possible, (III)actual.

These does NOT mean '*schizophrenia*', but represent students' rich mathematical actions, which are affected and co-evolve with their surrounding environment.

FUTUER TASK: Theorizing <u>mathematical working space</u> to take classroom complexity into consideration from the enactivist perspective

Thank you for your attention!